Hybrid Local Search Polynomial-Expanded Linear Multiuser Detectors for SIMO DS/CDMA Systems

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Abstract

In order to reduce computational complexity inherent to recurrent cross-correlation matrix inversion in a single-input-multiple-output (SIMO) direct-sequence code division multiple access (DS/CDMA) system, this work proposes a hybrid multiuser detector based on polynomial expansion (PE-MuD) followed by a low complexity local search procedure, aiming at obtaining a near-optimum multiuser bit-error-rate (BER) performance, but with an amount of computational processing saving time. The proposed hybrid PE-LS-MuD receiver topology suitable for multiple antennas DS/CDMA systems is analyzed under realistic wireless mobile channels, as well as useful system operation scenarios. Simulations results have indicated an improvement in performance-complexity trade-off regarding the classical linear multiuser detectors (LMuDs) performance, particularly, the mean square error minimization-based detector (MMSE).

Keywords: Near-optimum search algorithms, polynomial-expanded multiuser detection, Gerschgorin circles, SIMO DS/CDMA, complexity reduction

1. Introduction

In a multipath channel communication environment, spatial diversity can be obtained by employment of multiple antennas at the receiver, provided that the antennas are sufficiently far apart that the channel gains between different antenna pairs can be assumed to be independent [1]. Multiple-input-multiple-output (MIMO) communication systems, as well known, propitiate either an improvement in the detection performance (system reliability) or a system capacity increasing. However, due to the combination of effects of interference on the signal between the antennas, as well as the possible correlation between the received fading signals, those associated diversity or multiplexing gains could be jeopardized.

In code division multiple access (CDMA) systems, the total use of the transmission channel capacity, regardless of the channel adopted, depends on the features of the detector utilized, which prevent the effects generated by the multiple access through non orthogonal code division, mainly, in the effectiveness of the receptor to mitigate the effects of multiple access interference (MAI), as well as to deal with the near-far ratio (NFR) effect.

Multiuser detection algorithms usually have very high computational complexity, which greatly limits their adoption. The optimal solution for the multiuser detection problem lies on the deployment of maximum likelihood (ML) detector, proposed in [2]. However, ML detector complexity is impractical in almost scenarios of interest. Hence, linear near-optimal MuDs, such as the Decorrelator and MMSE were proposed in [3], [4]. Basically, these detectors utilize the inverse cross-correlation matrix of signature waveforms of the active users in the system ($R^{-1}$) to decouple the desired user's signal.

Aiming at more efficient linear detectors implementation, a multiple stage detection scheme, which approximately implements the inverse cross-correlation matrix through polynomial expansion in $R$, has been presented in [5]. The resulting detection scheme is namely polynomial-expanded multiuser detector and may be applied to approximate both the Decorrelator and minimum mean-squared error (MMSE) MuD algorithms. Polynomial-expanded multiuser detector can be viewed as an iterative approach in order to approximate the linear multiuser detectors with low complexity quadratic order dependence regarding the number of users, $O(K^2)$. In general, PE approach approximates the cross-correlation matrix inversion via Neumann iterative series expansion, with its coefficients estimated by the Gerschgorin circles method [6].

Other concept widely adopted in this current study is the local search (LS) based on neighborhood signal detection application. The LS detection method, which implement low-complexity local...
search solutions into a previously established neighborhood [7]. The main advantage of this method lies on its inexpensive very reduced complexity. According to [8], the LS-MuD have similar performance when comparable to the classical heuristic methods such as particle swarm optimization (PSO) and genetic algorithm (GA) algorithms, but with a convergence more accentuated which results in a smaller computational complexity. However, when the modulation order increases, such as deploying \( M \)-QAM with \( M \geq 16 \), the LS-MuD suffers with a lack of diversity in the search space, and the near-optimum performance achieved under low order modulation formats is deteriorated.

Several linear PE-MuDs algorithms aided or not by low complexity LS mechanisms in order to improve the performance-complexity trade-off of the linear MuDs have been proposed in the last decade, for instance in [9-15].

A PE-MuD with low complexity \( O(K^2) \) is presented in [9], which is able to reduce the computational burden related to the matrix inversion with \( O(K^3) \) necessary to implement the linear Decorrelator and MMSE multiuser detectors. In order to accelerate convergence, a normalized PE detector's matrix with respect to its smallest and largest eigenvalues is deployed. An efficient method to accurately estimate these eigenvalues is proposed for the first time. In [10], the same low complexity \( O(K^2) \) iterative approximation for the MMSE-MuD based on polynomial expansion is used. A new method based on the estimation of the eigenvalues of the channel correlation matrix by the implicitly restarted Lanczos method (IRLM) was suggested. As a result, the tight estimate of the eigenvalues has propitiated a better near-far resistance conjugated with a faster convergence when compared to the MMSE-MuD.

In [11], an iterative PE detector with faster convergence and better performance when compared to the MMSE-MuD is obtained even under high mobility scenarios.

A structure formed by the polynomial-expanded detector as the first stage followed by a local search algorithm has been presented in [12]. This structure is able to offer performance improvements under DSP implementable low-complexity perspective. In a same perspective, [14] investigate a new local search algorithm, which maintains the same convergence shape but with a smaller quantity of operations at the expense of a marginal and acceptable increasing in the BER. [14] introduces for the first time a hybrid detector that consists in the polynomial-expanded MMSE detector followed by a new hybrid local search strategy algorithm in a single-input-single-output (SISO) DS/CDMA system.

In [15] a hybrid multiuser detector based on polynomial expansion (PE-MuD) with \( \Phi \)-estimation aided by Gerschgorin circles, followed by a local search algorithm 1-adapt LS (one-adaptive local search), is proposed for SISO DS/CDMA systems with the name of hybrid PE 1-adapt-LS-MuD. Simulations results indicate that this structure achieves a near-optimum multiuser bit-error-rate (BER) performance, but with a considerable saving in computational complexity.

Besides this introductory Section, this work is divided in the following Sections. The system model is established in Section 2, in which a review on classic linear single-user (SuD) and multiuser detectors for SIMO DS/CDMA systems is presented. The polynomial expansion method deployed in order to approximate the cross-correlation matrix inversion is discussed in Section 3. The LS method applied to the MuD SIMO DS/CDMA problem is addressed in Section 4. Computational complexity is addressed in Section 5. Numerical results for BER performance and complexity of relevant MuD methods are unveiled in Section 6, while the main conclusions are pointed out in Section 7.

2. System model

Herein, a discrete-time baseband system model is adopted, with transmission through an uplink single-input multiple-output (SIMO) channel, i.e., a single antenna at the mobile terminal (MT) transmitters and \( N \) antennas at the base-station (BS) receiver side, subjected to additive white Gaussian noise (AWGN) and flat Rayleigh fading. The same channel is simultaneously shared by \( K \) users, which operate under a synchronous DS/CDMA system with binary phase shift keying (BPSK) modulation. This is equivalent a \( K \times N \) MIMO system. In the transmission, the \( i \)th information bit generated by the \( k \)th user, at a ratio of \( R_b = \frac{1}{T_b} \) bits per second is denoted by \( b_k[i] \). At each \( i \) bit interval, \( b_k[i] \) is modulated by a spreading sequence with pseudo-noise (PN) distribution and length \( L \). The spreading code can be represented by the vector:

\[
\mathbf{s}_k = [s_{k;1}, s_{k;2}, \ldots, s_{k:L}]^T
\]

(1)
where $s_k$: $\{0, 1\}$ and herein $L$ denoting the system's processing gain, i.e., the ratio between the bit information period and chip period, $L = \frac{T_b}{T_c} = \frac{R_c}{R_b}$, with $R_b$ and $R_c$ been the bit and chip rate, respectively. $(\cdot)^T$ denotes the matrix transposing operator.

Take into account a scheme of detection that explores the diversity gain, which is given by the utilization of $N$ multiple antennas at the base station (BS), the $L \times 1$ received signal vector at the $i$th bit interval and the $n$th receive antenna is represented by:

$$r_n \equiv \sum_{k=1}^{K} s_k \cdot c_{n,k} \cdot A_k b_k + w_n,$$

(2)

where $A_k$ is the amplitude of the transmitted signal by the $k$th user, admitted constant across the entire message; $w_n$ is the complex AWGN vector of the $n$th antenna, with mean zero and variance $\sigma_w^2 = N_0$, with bilateral power spectral density of AWGN noise given by $N_0=2$ W/Hz.

The term $c_{n,k}$ denotes the complex coefficient of the channel inherent to the $k$th user, at the $i$th bit interval, which corresponds to received signal at $n$th antenna and is perfectly known by the receptor, but not at the transmitter side. In statistical terms, $c_{n,k}$ may be represented by a circularly symmetric complex Gaussian random variable, with mean zero and variance $\sigma_c^2$ in the form $CN(0; \sigma_c^2)$. In the polar form, the channel's complex coefficient is described by:

$$c_{n,k} = |c_{n,k}| e^{j\mu_{n,k}},$$

(3)

where phase $\mu_{n,k}$ is uniform over the range $[0: 2\pi)$, i.e., omnidirectional BS receive antenna, and independent of the magnitude $|c_{n,k}|$. Herein, a non-line-of-sight (NLOS) communication has been assumed; hence, the magnitude of the channel coefficients is suitably characterized by a Rayleigh random variable [1] with probability density function given by:

$$f(r) = \frac{r}{\sigma_c^2} e^{-r^2/2\sigma_c^2}; r > 0.$$

(4)

In the notation of matrices, with bold capital letters representing matrices and bold lower case letters representing vectors, and suppressing the bit interval index $i$ for the sake of convenience, Eq. (2) could be rewritten as follows:

$$r_n = S C_n A b + w_n,$$

(5)

with $A = \text{diag}(A_1; A_2; \ldots; A_K)$ being the diagonal matrix of the transmit signal amplitudes, $S = [s_1; s_2; \ldots; s_K]$ is the spreading code matrix with dimensions $L \times K$ and $C_n = \text{diag}(c_{n,1}; c_{n,2}; \ldots; c_{n,K}) = \text{diag}([c_{n,1}; \ldots; c_{n,K}]) = F_n P_n$ corresponding to the channel complex coefficients matrix, where $F_n$ and $P_n$ are, respectively, the diagonal matrices of magnitudes and phases of the channel. Vector $b = [b_1; b_2; \ldots; b_K]^T$ contains bit information transmitted by the $K$ users and $w_n = [w_{n,1}; w_{n,2}; \ldots; w_{n,L}]^T$ is the complex noise vector with distribution $N(0; \frac{\sigma_w^2}{L})$.

The output signal of the conventional matched filters bank (MFB) is described taking account the channel phases:

$$y_{n}^{\text{mf}} = P_n^T r_n,$$

$$= S^T S C_n A b + P_n^T w_n,$$

$$= R F_n A b + w_n,$$

(6)

where the vector $y_n = [y_{n,1}; y_{n,2}; \ldots; y_{n,K}]^T$ represents the despread baseband-received signal, whose components are given by $y_{n,k} = S_k^T r_n$ and $y_{n,k}^{\text{mf}} = \sum_{i} y_{i,n,k}$ is the MFB output information vector; the cross-correlation matrix of the signature waveforms is obtained via $R = S^T S$;
vector \( \mathbf{w}_n = \mathbf{P}_n \mathbf{S}^T \mathbf{w}_n \) corresponds to the filtered noise with variance \( \frac{1}{n^2} \mathbf{R} \); the conjugate operator is denoted by \( (\cdot)^\dagger \).

Finally, the \( k \)th user's information bit is estimated by linearly combining the received signals from the \( N \) antennas:

\[
\mathbf{b}^{\text{est}}_k = \operatorname{sgn} \left< \sum_{n=1}^{N} \mathbf{y}^{\text{ref}}_{n,k} \right> ;
\]

where \( \operatorname{sgn}(\cdot) \) represents the signum function and \( <f \cdot g> \) is the real part operator. As a result, the estimated information bits vector is obtained as \( \mathbf{b}^{\text{est}} = [\mathbf{b}^{\text{est}}_1; \mathbf{b}^{\text{est}}_2; \ldots; \mathbf{b}^{\text{est}}_K]^T \). However, as well known, the performance of the Conventional detector decreases remarkably when the system loading \( L = \frac{K}{N} \) grows, i.e., due to the MAI level increasing as a function of the number of active users.

### 2.1. Optimum detection

The optimum performance is obtained with the use of the ML detector, presented in [2]. ML detector performs the joint information detection of the \( K \) users in the system, maximizing the following cost function:

\[
\mathcal{D}_n(b) = 2 < b^T F_n A y^{\text{ref}}_n b > - b^T C_n A R A C_n^H b ;
\]

which is based on the Euclidean distance between the received signal at \( n \)th antenna and the signal reconstructed in the receptor from the information candidate vector, \( \mathbf{b} \); the matricial operator \( (\cdot)^H \) holds for conjugation and transposition.

The optimum multiuser detection (O-MuD) criterion yields the best information bits estimated vector \( \mathbf{b}^{\text{opt}} \):

\[
\mathbf{b}^{\text{opt}} = \underset{b \in \mathbb{B}_2^M}{\arg \max} f(b) ;
\]

where \( \mathbb{P} \) is the transmitted message length and \( M \) the symbol alphabet dimension; \( f(b) \) denotes the linearly combining cost function, given by:

\[
f(b) = \frac{1}{N} \sum_{n=1}^{N} \mathcal{D}_n(b) ;
\]

For the binary modulation adopted in this work, \( M = 2 \). Although the optimum ML detector achieves the best performance, however, its computational complexity is exponential with the number of users, modulation order \( m \) and message length, i.e., it is of \( \mathcal{O}^{2^M} \). As a result, huge amount of suboptimum and near-optimum multiuser detectors have been proposed in the last two decades [2], [16].

### 2.2. Linear methods of multiuser detection

In [3], linear methods of detection were discussed, including the Decorrelator detector. This one operates from multiplication of the discrete signals at the matched filters output by the inverse cross-correlation matrix \( R^{-1} \). Considering the coherent reception model, the information bits vector at the \( r \)th antenna, which is estimated after the application of the Decorrelator multiuser filter, can be conveniently described as follows:

\[
\mathbf{z}^{\text{dec}}_n = R^{-1} y^{\text{ref}}_n = R^{-1} \mathbf{F}_n A \mathbf{b} + R^{-1} \mathbf{w}_n ;
\]
where $T_{\text{dec}} = R^{-1}; 8 n$ is the transformation matrix for the MuD Decorrelator filter, and $z_{\text{dec}} = [z_{\text{dec}}^{-1}; z_{\text{dec}}^{-2}; \ldots; z_{\text{dec}}^{-K}]^T$ is the Decorrelator output information vector. Note that the cross-correlation inverse matrix $R^{-1}$ is the common filter for all antennas' received signals. The information bit of the $k$th user, obtained at Decorrelator detector output, is estimated by the linear combining of the corresponding $k$th received signals at all $N$ antennas:

$$b_{\text{dec}}^{k} = \text{sgn}(\text{sign} \left( \sum_{n=1}^{N} z_{\text{dec}}^{-n;k} \right))$$

(12)

The Decorrelator detector presents a gain in the performance regarding the Conventional detector, although the power associated to the noise term $\psi_{n} = R^{-1}P_{S}S^{T}w_{n}$, obtained at the Decorrelator output, is always higher or equal to the noise term obtained at the Conventional output [2], [17].

Another classical linear detection method known in literature is the MMSE detector, proposed for CDMA systems in [4]. This method is based on the appropriate choice of a linear transformation vector, $t_{n;k} = [t_{n;1}; t_{n;2}; \ldots; t_{n;K}]^T$, that minimizes the mean square error (MSE) between the $k$th user's information bit and the $k$th linear transformation output, $t_{n;k}y_{n}f^{b}$, resulting in:

$$t_{\text{mmse}}^{k} = \min_{t_{n;k}} E_{n}(b_{k} - t_{n;k}y_{n}f^{b})^{2}$$

(13)

The vector that minimizes (13) involves the covariance of colored noise $\psi_{n}$ and the estimated amplitudes of the received users' signals at the $n$th BS receive antenna, expressed by the $K \times K$ diagonal matrix:

$$G_{n} = F_{n}A$$

(14)

By applying this MMSE solution to the joint detection of the $K$ users, the transformation matrix $T_{\text{mmse}}^{n} = [t_{\text{mmse}}^{n;1}; t_{\text{mmse}}^{n;2}; \ldots; t_{\text{mmse}}^{n;K}]^T$ with dimension $K \times K$ is given by:

$$T_{\text{mmse}}^{n} = R + \frac{1}{2}G_{n}^{1/2}$$

(15)

Hence, the decision information vector at the $n$th antenna, which is obtained after the application of the MMSE multiuser filter, is described by:

$$x_{\text{mmse}}^{n} = T_{\text{mmse}}^{n}y_{n}f^{b}$$

where $z_{\text{mmse}}^{n;k} = t_{\text{mmse}}^{n;k}y_{n}f^{b}$. Therefore, the estimated information bit for the $k$th user at the output of the linear SIMO BPSK MMSE detector is obtained as a linear combination of the decision variable on the $N$ antenna branches:

$$b_{\text{mmse}}^{k} = \text{sgn}(\text{sign} \left( \sum_{n=1}^{N} x_{\text{mmse}}^{-n;k} \right))$$

(16)

3. Polynomial-expanded multiuser detectors

The computational complexity of the linear MuDs, which originates in the operations associated to the cross-correlation matrix inversion, grows with the third order of the matrix size, i.e., $O^\left( mPK \right)^3$. However, any linear transformation matrix, represented by $T_{n}$ in the SIMO model context, can be approximated through the iterative polynomial expansion method with complexity of $O^\left( mPK \right)^2$.

3.1. General result for PE matrix approximation

The general result for the $K \times K$ polynomial-expanded transformation matrix $T_{\text{pe}}$, which is able to implement a PE multiuser detector (by approximating a matrix inversion), is given by [17]:

$$T_{\text{pe}} = \left[ w_{i}Q^{i} \right]_{i=0}^{\infty}$$

(17)
where $N_t$ denotes the number of terms of the polynomial expansion. The matrix $Q$ and the weights $w_i$, interpreted as the coefficients for the series convergence rate, have to be chosen such that they suitably approximate the desired multiuser detector. As a result, the polynomial expansion transformation matrix $T_{pe} = Q^{-1}$ when the number of expansion terms $N_t \rightarrow 1$.

### 3.2. Polynomial expansion via Neumann series

By using the Neumann series expansion method [6], the inverse cross-correlation matrix $R^{-1}$, for the case of Decorrelator, may be approximated as:

$$R^{-1} \approx T_{dec} = \sum_{i=0}^{\infty} (I_K - \mathcal{R})^i$$  \hspace{1cm} \left( R < 1 \right) \quad (18)$$

where $I_K$ is an identity matrix of size $K$, and the associated residual error matrix is given by:

$$*_{dec} = \sum_{i=N_t+1}^{\infty} (I_K - \mathcal{R})^i$$  \hspace{1cm} \left( -1 \right) \quad (19)$$

such that the equality $R^{-1} \approx T_{dec} + *_{dec}$ holds.

In the same way, the PE expansion matrix transformation for the SIMO linear MMSE detector at the $n$th receive antenna can be approximated in:

$$T_{n,mmse} = \sum_{i=0}^{\infty} (I_K - \mathcal{R} + \mathbf{G}_n^2)^i$$  \hspace{1cm} \left( \mathbf{z}_{\text{pe}}^n < 1 \right)$$

3.3. Convergence factor

In Eq. (18), the convergence factor of the Neumann series is equal to the spectral radius$^1$ of the matricial operator, $\| (I_K - \mathcal{R}) \|$. Therefore, the series converges if the value of the spectral radius is less than one [18]. Assuming that the eigenvalues of the $K \times K$ matrix $\mathcal{R}$ are $\lambda_k$, $k = 1; 2; \cdots; K$, all real and limited to the interval:

$$\lambda_{\text{min}} \leq \lambda_k \leq \lambda_{\text{max}};$$

the eigenvalues of $(I_K - \mathcal{R})$, namely $\lambda_k - 1$, with $k = 1; 2; \cdots; K$, will be laid in the interval:

$$1 \leq \lambda_k \leq 1 + 1/\lambda_{\text{max}};$$

Also assuming that $\lambda_{\text{min}} > 0$, the convergence for the Neumann series depends on the following conditions:

$$1 \leq \lambda_{\text{min}} < 1;$$

$$1 \leq \lambda_{\text{max}} > 1;$$

As a consequence, the series converges with any scalar $\mathcal{R}$ which satisfies:

$^1$ Spectral radius of a matrix corresponds to the absolute value of its greater eigenvalue.
To simplify the analysis for the case of linear MMSE SIMO DS/CDMA detector, the convergence factor is defined herein with the premise that the diagonal received-amplitudes matrix at the \( n \)th receive antenna is normalized as \( G_n = I_K \cdot B_n \). Otherwise, the upper bound of \( \Phi \) would be changed for each different value of \( G_n \) entries. Thus, the condition for the convergence of the series is:

\[
\Phi |I_K - \Phi R + \Phi^2 K| < 1
\]

Therefore, the parameter \( \Phi \) that determines the convergence of the PE-MMSE SIMO detector is found on the interval:

\[
0 < \Phi < \frac{2}{j \cdot \max + \frac{2}{j \cdot \max}}
\]

### 3.3. Optimum value of the parameter \( \Phi \)

Since the convergence factor of an iterative method can be associated with the radius of the matricial operator, the convergence ratio is related to the dimension of this radius [18]. The spectral radius for the matricial operator which approximates the Decorrelator filter is given by:

\[
\frac{1}{j} (I_K - \Phi R) = \max \{ j | \Phi_{\max} j, j | \Phi_{\min} j \}
\]

Therefore, the best convergence rate is obtained with the proper choice of the scalar \( \Phi \), in order to optimize the spectral radius. Fig. 1 shows the behavior of the spectral radius as a function of \( \Phi \). Hence, it follows that the optimum value of \( \Phi \) is found between the zeros of the bounds of the intervals in (24), at the point defined by the intersection between the positive slope of the lower bound curve with the negative slope of the upper bound curve [18]:

\[
j \cdot 1 + \Phi_{\max} = 1 \cdot j \cdot \Phi_{\min}
\]

This operation results in the optimum parameter for linear Decorrelator detector in the PE approximation defined by:

\[
\Phi_{\text{opt}}^{\text{DEC}} = \frac{2}{\Phi_{\min} + \Phi_{\max}}
\]

In turn, for the linear MMSE detector approximation, the optimum value of \( \Phi \) is given by:

\[
\Phi_{\text{opt}}^{\text{MMSE}} = \frac{2}{\Phi_{\min} + \Phi_{\max} + 2 \cdot \Phi_{\max}}
\]

Important to point out that the deterministic choice of \( \Phi_{\text{opt}} \) through the eigenvalues calculation of the cross-correlation matrix is prohibitively complex for the implementation of the polynomial expansion method using practical digital signal processing hardware platforms. As a result, the complexity of only one eigenvalue computation, as well as of all eigenvalues calculation from a \( K \) squared-dimension matrix results in \( O \cdot K^3 \). Thus, it is necessary to estimate the optimum value of the parameter \( \Phi \). Next, the estimation of \( \Phi_{\text{opt}} \) is suggested by using the Gerschgorin circles Theorem [6].

### 3.4. Gerschgorin circles Theorem

According to Gerschgorin theorem, any eigenvalue \( \lambda \) of a matrix \( R \), which has elements \( r_{i,j}, B_i; j \), is situated in one of the circles of the complex plane that are centered in \( r_{i,i} \), with radius \( |r_{i,i}| |r_{i,j}| \), i.e.,

\[
j |r_{i,i}| \cdot \cdots \cdot |r_{i,j}| \cdot \cdots \cdot j |r_{i,j}|
\]

\[
j |B_i| |r_{i,i}| \cdot \cdots \cdot |r_{i,j}| \cdot \cdots \cdot j |B_i|
\]
Thus, through a simple calculation, by using the elements of \( R \), the approximated values of \( \lambda_{\min} \) and \( \lambda_{\max} \), which are denoted by \( \lambda_{\min} \) and \( \lambda_{\max} \), respectively, can be achieved by:

\[
\begin{align*}
\lambda_{\min} & = \min_{i} \left( \frac{1}{\lambda_{\max}} \left( r_{i;i} + \sum_{j \neq i} |r_{i;j}| \right) \right) ; \quad \text{8i}; \\
\lambda_{\max} & = \max_{i} \left( \frac{1}{\lambda_{\max}} \left( r_{i;i} + \sum_{j \neq i} |r_{i;j}| \right) \right) ; \quad \text{8i}.
\end{align*}
\]  

(28)  

(29)

The Gerschgorin circles Theorem (GCT) guarantees a considerable reduction in the complexity of the calculation of the minimum and maximum eigenvalues; therefore, in this work GCT results are adopted in order to estimate the \( \rho \) parameter.

4. Local search methods applied to the multiuser detection

Local search methods propitiate the attainment of near-optimum solutions from searches guided in subspaces of the dimension of the optimization problem. The local search algorithm 1-opt LS [12], [19] is discussed in the following; after that, an adaptation for the 1-opt LS algorithm is proposed in the Subsection 4.2; a new algorithm for local guided search applicable to the SIMO BPSK multiuser detection is described the Algorithm 2.

4.1. Local search algorithm 1-opt LS

The deterministic search algorithm one-optimum local search (1-opt LS) performs guided searches for the vector that maximizes the linearly combining cost function posed by (10), selecting candidate-vectors located inside the unitary Hamming distance\(^2\) from the output vector of MFB. Herein, the users' power profile is considered for classification purpose at the beginning of the guided search process. From the estimated users' received amplitudes diagonal matrix \( G_n = F_n A = \text{diag}(g_{n;1}, g_{n;2}, \ldots, g_{n;K}) \), the average received amplitude of the \( k \)th user can be estimated as a linear combination of the corresponding \( N \) antenna branches:

\(^2\) Hamming distance between two vectors, e.g., \( b_1 \) and \( b_2 \), is defined by \( d_h(b_1; b_2) = k_{b_1} \iff b_2 \), which corresponds to the amount of elements that differ between the vectors.
where vector \( g = [g_1; g_2; \ldots; g_K]^T \) is formed to assist the users' power classification. The pseudo-code for the local search algorithm 1-opt LS is described in the Algorithm 1.

### 4.2. Local search algorithm 1-adapt LS

The quantity of calculations of the cost functions during the search for the best candidate vector can be limited by using a given threshold. Chase establishes a threshold criterion based on channel measurement information, by selecting a fixed number of the lowest confidence bits to be changed [20]. Differently of Chase search stop criterion, herein for the proposed 1-opt LS algorithm, a dynamic threshold is used to create adaptation and reduce complexity. This new algorithm, namely 1-adapt LS (one-adaptive local search), classifies the received signals in order of increasing amplitude. Then, candidate vectors with unitary Hamming distance are generated, following the ordering of the signals (from the weakest to the strongest), and their respective cost functions are evaluated. In case of the linearly combining cost function value does not increase inside of a pre-established quantity of consecutive evaluations, denoted here by parameter \( \cdot \), the search process is interrupted and a new search is initiated. In general, \( \cdot \) is taken as a fraction of the number of active users in the system. The pseudo-code for the algorithm 1-adapt LS is described in the Algorithm 2.

#### Algorithm 1. One-optimum LS

Input: \( \mathbb{B}^{mt}; N_{it}; g; \)  
Output: \( \hat{b}; \)  

\[
\begin{align*} 
\text{begin} \\
\quad t = 0; \\
\quad \text{1. Classify signals: } g \text{ (increasing amplitude order), given } g_k[t], k = 1; 2; \ldots; K \text{, with } g_k[t] \cdot g_{k+1}[t]; \\
\quad \text{2. Initialize the local search: } t = 1; \\
\quad \quad \text{b}^{best}[1] = \mathbb{B}^{mt}. \\
\quad \text{3. for } t = 1; 2; \ldots; N_{it}, \\
\quad \quad \text{a. Generate candidate vectors with unitary Hamming distance} \\
\quad \quad \quad \text{denoted by } b_i[t], i = 1; 2; \ldots; K; \\
\quad \quad \text{b. Calculate } f(b_i[t]); \\
\quad \quad \quad \text{c. if } 9b_i[t]; (j \neq i); \[ f(b_j[t]) > f(b^{best}[t]) \]^\top [f(b_j[t]) > f(b^{best}[t])]; \text{,} \\
\quad \quad \quad \text{else, go to step 4} \\
\quad \quad \text{end if} \\
\quad \text{end for} \\
\quad \text{4. } \hat{b} = b^{best}. \\
\text{end} 
\end{align*}
\]

When the Algorithm 1-adapt LS prioritizes the inversion of the weakest signals' information vector, assuming that these signals have a greater error probability in the reception, the effectiveness of the search for the best candidate vector is potentiated. Thus, it is possible to reduce the complexity of the detector by limiting the number of signals processed, when the relative increasing in SNR values across the iterations indicates the stagnation in the cost function gain. This method results in a decreasing of the average quantity of cost function calculations by iteration, denoted herein by \( \bar{\sigma}^{avg} \), but with a small performance degradation of the detector.

### 4.3. Hybrid local search polynomial-expanded MuDs

In DS/CDMA multiuser detection, polynomial-expanded matrix inversion approximation is combined to search mechanisms in order to obtain a better trade-off between complexity and near-optimum performance. We propose a hybrid structure in which an adaptive subspace search is implemented, as an improvement to the multiuser detector proposed in [12].
4.3.1. Hybrid PE-MMSE 1opt-LS-MuD detector

A detection structure formed by a suboptimal local search algorithm in conjunction with a primary stage of polynomial-expanded linear multiuser detector was presented in [12]. This structure has been reproduced herein, by deploying in the first stage, the polynomial-expanded MMSE detector with $N_t$ terms and $\Phi$ estimated via the Gerschgorin circles method; in the second stage it is followed by a 1-opt local search described in Algorithm 1. Hereafter it is identified as hybrid PE-MMSE 1opt-LS-MuD.

4.3.1. Hybrid PE-MMSE 1adapt-LS-MuD detector

We introduce for the first time a multiuser detector constituted by the polynomial-expanded MMSE detector followed by a new local search algorithm 1-adapt LS based on Algorithm 2, hereafter named the hybrid PE-MMSE 1adapt-LS-MuD, with $N_t$ terms and $N_{it}$ iterations. In Subsection 6.1 a performance comparison including both hybrid suboptimal multiuser detectors has been carried out.

Algorithm 2. One-adaptive LS

\begin{verbatim}
Input: $B^{mf}$; $N_{it}$; $g$; \cdots 

Output: $B$

begin

$t = 0$

1. Classify signals: $g$ (increasing amplitude order), given $g_k$, $k = 1; 2; \cdots; K$, with $g_k \cdot g_{k+1}$

2. Initialize the local search: $t = 1$; ` = 0

$B_{best}[1] = B^{mf}$

$f_{best}[1] = f(B^{mf})$

3. for $t = 1; 2; \cdots; N_{it}$

while ` < ·

a. Generate candidate vectors with unitary Hamming distance denoted by $b_i$, $i = 1; 2; \cdots; K$

b. Calculate $f_i = f(b_i)$

if $f_i > f_{best}[t]$ then

$b_{best}[t+1] = b_i$

$f_{best}[t+1] = f(b_{best}[t+1])$

` = 0;

else

` = ` + 1;

end if

end while

if $f_{best}[t+1] = f_{best}[t]$, go to step 4

end if

end for

4. $B = b_{best}$

end

end

5. Computational complexity

The computational complexity metric is defined as the total number of floating point operations needed for each detector to achieve the convergence. The considered operations are: multiplication, comparison and random number generation. The complexity is expressed as a function of the number of users ($K$), receive antennas ($N$) and iterations needed for convergence ($N_{it} \cdot N_{it}$). The value of the computational complexity of the 1adapt-LS-MuD is expressed as a function of the average quantity of cost function calculations by iteration $3_{avg}$, since the quantity of cost function calculations is not a constant through the iterations loop, SNR and MAI levels.

It is worth noting that the cost function calculation in (8) is the most significant factor in determining the complexity of the detectors. The terms $F_N A y^T_{mf}$ and $C_n A R A C_n$ are evaluated outside the iterations loop and adopted constant during the detector guided search. The resulting number of operations needed for these two terms is $3K^3 + 4K^2$, and this calculation is done $N$ times.
(one for each antenna). Inside the iterations loop, the number of operations needed for each candidate vector evaluation through cost function becomes $N \cdot 3K^2 + 2K$. Table 1 depicts the computational complexity for the PE-MMSE, 1opt-LS-MuD and 1adapt-LS-MuD stages, as well as for the overall ML detector.

Table 1. Computational complexity in [flops]

<table>
<thead>
<tr>
<th>MuD</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>$2^K \cdot N(3K^2 + 2K) + N(3K^3 + 4K^2)$</td>
</tr>
<tr>
<td>1opt-LS n1</td>
<td>$n_1K(3K^2 + 2K) + N(3K^3 + 4K^2 + 2K) + 1$</td>
</tr>
<tr>
<td>1adapt-LS n1</td>
<td>$n_1P_{avg}N(3K^2 + 2K) + 3K + 3j\cdot N(3K^3 + 4K^2 + 2K) + 1$</td>
</tr>
<tr>
<td>PE-MMSE</td>
<td>$N(3K^2 + K(N^2 + 2) + 4)$</td>
</tr>
</tbody>
</table>

6. Performance-complexity analysis

In this Section, the performances of the suboptimal MuDs are evaluated by means of Monte Carlo simulation (MCS) method. The flat Rayleigh fading channels with magnitude and phase coefficients perfectly estimated at the receptor side have been assumed, while the number of antennas at receptor side has been fixed in $N = 4$ antennas. In all numerical results presented in this Section the average SNR, denoted by $\text{SNR}_{avg}$, is deployed in the context of the near-far effect, i.e., there are two interfering groups of users with near-far ratio ($NFR$):

$$NFR^{(I)} = \frac{P_{\text{interf}}}{P_{\text{interest}}} = +5 \text{ dB (}K=3 \text{ users)};$$

$$NFR^{(II)} = \frac{P_{\text{interf}}}{P_{\text{interest}}} = -5 \text{ dB (}K=3 \text{ users)},$$

where $P_{\text{interf}} = a_{\text{interf}}^2$ and $P_{\text{interest}} = a_{\text{interest}}^2$ represent the received power related to the linear combination of received amplitudes, Eq. (30), for the $i$th interfering signal and $j$th interest signal, respectively.

A variant of the near-far effect considering only one interfering group of users with:

$$NFR^{(I)} = \lceil 15: 30 \text{ dB (}K=2 \text{ users)}$$

is deployed in the MCS; numerical results for the $NFR^{(I)}$ condition are depicted in Fig. 4. Hence, the average SNR and bit-error-rate ($\text{BER}_{avg}$) presented in this Section have been evaluated just over the interest users group only ($K=3$ or $K=2$ users). The $\text{SNR}_{avg}$ considered in simulations ranges from 0 to 20 dB. Random spreading codes with processing gain length of $L = 31$ have been adopted in a single-rate DS/CDMA system. Furthermore, the number of terms in polynomial expansion is limited to $N_t \leq f1; 3g$ terms, while the number of iterations of the local search algorithm is limited to $N_t \leq f1; 3; 5g$ iterations.

As one can note through Fig. 2, the 1adapt-LS-MuD with 5 iterations or more is robust to the growth of the system loading, as well as the 1opt-LS-MuD. Taking into account $N_t = 1$ iteration, the $\text{BER}_{avg}$ performance of both evaluated local search detectors are very close. Nevertheless, the complexity of the 1adapt-LS-MuD detector is smaller, according to Fig. 3. By loading the system in approximately 90%, i.e., $K = 27$ users, the value of $\text{SNR}_{avg}$ accomplished for 1adapt-LS-MuD detector...
with one iteration is 30% smaller than 1opt-LS-MuD detector, although with no increasing in the BER$_{\text{avg}}$, as one can see in Fig. 2.

![Figure 2. System loading robustness performance for 1opt-LS-MuD and 1adapt-LS-MuD detectors. Flat Rayleigh channel and SNR$_{\text{avg}}$ = 10 dB; N = 4 antennas and Algorithm 2 with $\cdot = \epsilon/6, \epsilon K$.](image)

6.2. Hybrid PE-MMSE 1adapt-LS-MuD detector

Fig. 4 shows the near-far robustness for the hybrid PE-MMSE 1opt-LS and 1adapt-LS-MuDs detectors with $L = 65\%$, in comparison with the linear MMSE and the PE-MMSE-MuDs. In this simulation, $K_{\text{inter}} = 10$ and $K_{\text{interest}} = 10$ users have been considered and the error probability is evaluated only for the interest group. One can see that combining PE-MMSE with the 1-opt LS and the proposed 1-adapt LS algorithms provides an improvement in terms of the BER performance. But the hybrid detectors are not completely robust against strong MAI configurations (NFR > 10 dB). For the
1-adapt LS algorithm, in the region of negative NFR, i.e., when the power of the received signal of the interest group is greater than the interfering group, the power classification mechanism has a negative influence on the BER performance of the interest group of users because the quantity of flips of the strongest users’ bits are reduced by the dynamic threshold $\cdot$. In this near-far ratio region, i.e., when $NFR < 0 \text{ dB}$, the detection could be switched from PE-MMSE 1adapt-LS-MuD to PE-MMSE 1opt-LS-MuD.

Fig. 4. Near-far robustness NFR for the hybrid PE-MMSE 1adapt-LS-MuD detector; Flat Rayleigh channel with $K = 20$ users; Local search algorithms with $N_{\text{it}} = 3$ iterations and Algorithm 2 with $\cdot = b0.6 \cdot K$; SNR$_{\text{avg}} = 10 \text{ dB}$ and $N = 4$ antennas.

Fig. 5 depicts the BER performance of the MuDs considered under SIMO BPSK environment for low system loading $L = 29\%$ ($K = 9$ users) configuration. In this scenario, the hybrid PE-MMSE 1opt-LS-MuD with $N_{\text{it}} = 3$ terms keeps the same diversity gain achieved by the linear MMSE-MuD but with a BER performance improvement, with a SNR gap of $\approx 2 \text{ dB}$ in high SNR region. On the other hand, the proposed hybrid PE-MMSE 1adapt-LS-MuD follows the performance of the PE-MMSE 1opt-LS-MuD in the low and medium SNR region; however it achieves BER floor at SNR$_{\text{avg}} > 20 \text{ dB}$. A reduction in BER floor effect can be obtained by adding more terms (5 or 7 terms), with marginal complexity increasing. Hence, the proposed hybrid PE-MMSE 1adapt-LS-MuD represents a suitable solution in terms of performance-complexity trade-off when compared to the traditional suboptimum linear PE-MMSE-MuD.

7. Conclusions

The proposed local search algorithm 1-adapt LS promotes a remarkable gain in the DS/CDMA system performance equipped with polynomial expansion-based hybrid multiuser detectors. When associated to low-complexity PE-MuD detectors, it provides reliability to the detection process, without an excessive increasing in its implementation cost, been able to offer a suitable performance-complexity trade-off.

Simulation results have shown that the proposed 1-adapt LS is able to provide a considerable level of robustness against the near-far effect when combined to the PE-MuD. Furthermore, the proposed hybrid multiuser detector presents a remarkable trade-off between near-optimum performance and reduced complexity, especially when the detector operates in scenarios with medium-high system loadings and moderate or low NFR.
Figure 5. Hybrid PE-MMSE 1adapt-LS-MuD detector performance, in the flat Rayleigh channel with $N=4$ antennas; Local search algorithms with $N_{it} = 3$ iterations and Algorithm 2 with $\epsilon = 0.6\cdot K$ for $K=9$ users.

8. Acknowledgement

This work was supported in part by the National Council for Scientific and Technological Development (CNPq) of Brazil under Grants 202340/2011-2, 303426/2009-8 and in part by Londrina State University - Paraná State Government (UEL).

9. References


